MODEL OF LIQUID FILM FLOW OVER A REDUCER SURFACE

L. I. Sen', A. M. Te,
and O. Yu. Tsvelodub

UDC 532.51

The regularities of film flow over reducer surfaces must be known to compute heat- and mass-transfer processes in expander film apparatus [1]. A two-dimensional laminar wave-free film flow regime is realized in a broad range of mass flow variation [2] in the gravitational runoff of the fluid over the reducer with a general aperture greater than 90° (in contrast to the flow over the vertical surface).

Let us consider the stationary axisymmetric flow of a thin fluid film over a reducer. In this case, the application of the Navier—Stokes and the continuity equations in the boundary-layer approximation is allowable in considering the problem. Let us introduce the following coordinate system: the y axis is perpendicular to the reducer generator and is directed upward, the x axis coincides with the reducer generator and is directed downward, and the origin is the reducer entrance edge (Fig. 1). These equations are written in the coordinate system chosen in the form [3]

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+g\sin\alpha; \tag{1}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y}-g\cos\alpha=0; \tag{2}$$

$$\partial(ur)/\partial x+\partial(vr)/\partial y=0, \tag{3}$$

where u and v are, respectively, the x and y components of the velocity, g is the free-fall acceleration, ν is the fluid kinematic viscosity, p is the pressure, r(x) is the distance between the axis and the reducer generator, and α is the slope of the reducer generator to the horizontal.

The problem is solved under the following boundary conditions

$$\text{for}\quad y=0\ u=0,\ v=0; \tag{4}$$

$$\text{for}\quad y=h(x)\ \tau=0\to\partial u/\partial y=0; \tag{5}$$

$$p=p_a; \tag{6}$$

$$udh/dx=v, \tag{7}$$

where p_a is the pressure over the free surface, and h(x) is the running thickness of the film.

Integrating (2) and using condition (6), we obtain an expression for the pressure

$$p=p_a+\rho g[h(x)-y]\cos\alpha. \tag{8}$$

From (1) and (8) we have

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{dh}{dx}\cos\alpha+\nu\frac{\partial^2 u}{\partial y^2}+g\sin\alpha. \tag{9}$$

Assuming the flow self-similar, i.e.,

$$u(x,\ y)=V(x)f(z),\ z=y/h(x), \tag{10}$$

we can integrate (9) and (3) with respect to the coordinate y between 0 and h(x). Experi-

---

Vladivostok and Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 49-52, May-June, 1983. Original article submitted May 13, 1982.

0021-8944/83/2403-0325$07.50

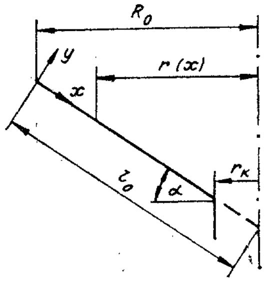

Fig. 1

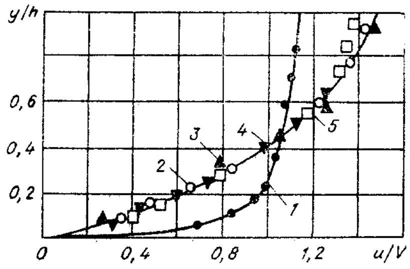

Fig. 2

mental data [2] on measurement of the distribution of the longitudinal velocity component in the film transverse section over the length of the transit show that the assumption (10) is satisfied sufficiently well on the fundamental section of the reducer (in Fig. 2 (α = 20°, Re = 120) the points 1-5 correspond to the values of x equal to 0, 15.5, 30.0, 53.5, 75.0 mm).

After simple manipulations, we obtain the following system of equations from (9) and (3) by using (5), (7) and (10):

$$\gamma \frac{d(V^2rh)}{dx} = -\beta \frac{V}{h} \nu + gh \sin\alpha - gh \frac{dh}{dx} \cos\alpha; \tag{11}$$

$$\frac{d(Vrh)}{dx} = 0. \tag{12}$$

Here

$$\beta = \frac{df}{dz}\Big|_{z=0}; \ \gamma = \int_0^1 f^2(z)\,dz.$$

In deriving the system (11) and (12) it was also assumed that V(x) is the mean mass flow rate, i.e.,

$$\int_0^1 f(z)\,dz = 1.$$

From (12) we have

$$V(x)r(x)h(x) = V_0 R_0 h_0, \tag{13}$$

where V_0, R_0, h_0 are, respectively, the mean mass flow rate, radius, and thickness of the film on the entrance edge of the reducer.

For a rectilinear profile of the reducer surface generator, the relation between the coordinate x and the running radius is determined by the formula

$$r(x) = (l_0 - x)\cos\alpha, \tag{14}$$

where $l_0 = R_0/\cos\alpha$ is the length of the cone generatrix.

By using (13) and (14), we obtain an equation to determine the film thickness h(x) from (11):

$$\frac{\gamma V_0^2 h_0^2 l_0^2}{(l_0 - x)^2}\left[\frac{1}{(l_0 - x)h} - \frac{h'}{h^2}\right] = \beta \frac{V_0 h_0 l_0}{(l_0 - x)h^2} \nu + gh\sin\alpha - gh\frac{dh}{dx}\cos\alpha. \tag{15}$$

Let us rewrite (15) in dimensionless form

$$[1 - (1 - x)^2 H^3 \cos\alpha/(\mathrm{Fr}\gamma)] \frac{dH}{dx} = \frac{H}{(1 - x)} + \frac{\beta(1 - x)}{\mathrm{Re}\,\varepsilon\gamma} - \frac{\sin\alpha}{\mathrm{Fr}\,\gamma\varepsilon}(1 - x)^2 H^3, \tag{16}$$

where $x = x/l_0$; $H = h/h_0$; $\mathrm{Fr} = V_0^2/gh_0$; $\mathrm{Re} = V_0 h_0/\nu$; $\varepsilon = h_0/l_0$ (the equation is valid for $\varepsilon \ll 1$).

As is seen from (16), the characteristics γ and β of the velocity profile must also be known in addition to the initial values of the velocity V_0 and the film thickness h_0 to solve it.

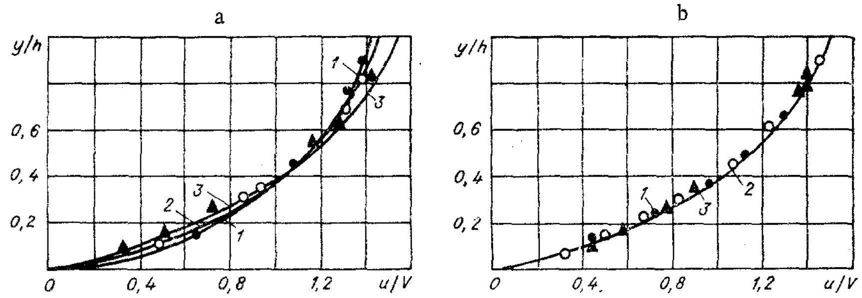

Fig. 3

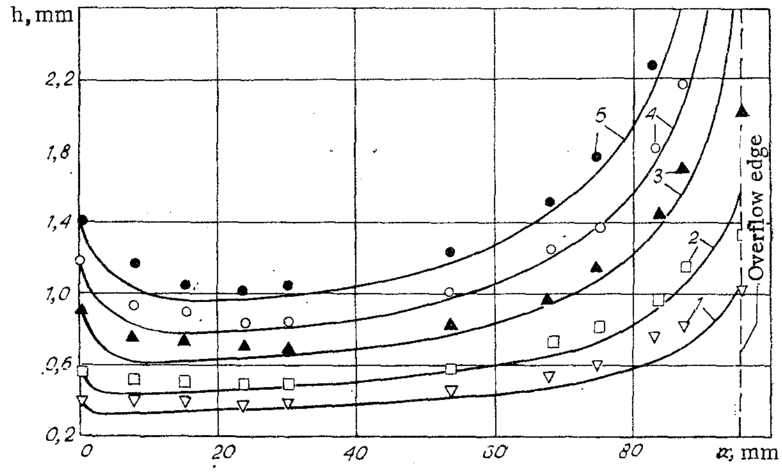

Fig. 4

The parameter γ characterizing the filling of the velocity profile, varies slightly (for instance, for a shock profile $\gamma = 1.0$, for a Poiseuille profile — 1.2). It is experimentally established [2] that in the investigated range of variation $\alpha = 10\text{-}45°$ and the liquid rate density $\Gamma_0 = 30\text{-}255\ \mathrm{mm}^2/\mathrm{sec}$, the quantity γ varies between the limits 1.12 and 1.19; hence, for the numerical solution of (16) the value of γ is taken equal to 1.15.

The quantity β characterizing the friction stress on the wall, grows as the Reynolds number increases (Fig. 3a) and depends weakly on the angle α. The velocity distribution over the film section is presented in Fig. 3a and b: a) for $\alpha = 30°$, x = 30 mm with Re = 30, 60, 120 (curves 1-3, respectively), and b) for Re = 120, x = 15.5 mm for $\alpha = 10, 20, 30°$ (the points 1-3, respectively). For the computations values of β were taken equal to their experimental values on the fundamental section of the reducer while V_0 and h_0 were taken equal to their experimental values [2].

A comparison of the results of solving (16) with the experimental data on the film thickness measurements [2] is presented in Figs. 4 and 5. In Fig. 4 for $\alpha = 20°$ $\gamma = 1.15$, Re and β are 30 and 3.0, 60 and 3.25, 120 and 3.5, 180 and 4.0, 240 and 4.75 (curves and points 1-5, respectively). In Fig. 5 Re = 120, $\gamma = 1.15$, $\beta = 3.5$ for $\alpha = 10, 20, 30, 45°$ (curves and points 1-4, respectively). Results of the comparison show that the equation describes all the characteristic flow section well.

The greatest deviation from the experimental data is observed at the entrance section to the reducer, where the velocity profile differs strongly from the self-similar (see Fig. 2); here the computed values lie below the experimental. This is explained by the fact that the experimental values of β in this domain are higher than taken in the computation, and since the solution of (16) in the neighborhood of the point x = 0 has the following form in a first approximation

$$H = 1 + \left(\frac{\beta}{\mathrm{Re}} - \frac{\sin\alpha}{\mathrm{Fr}}\right)\frac{x}{\gamma\varepsilon},$$

it is clear that the theoretical values of h decrease rapidly.

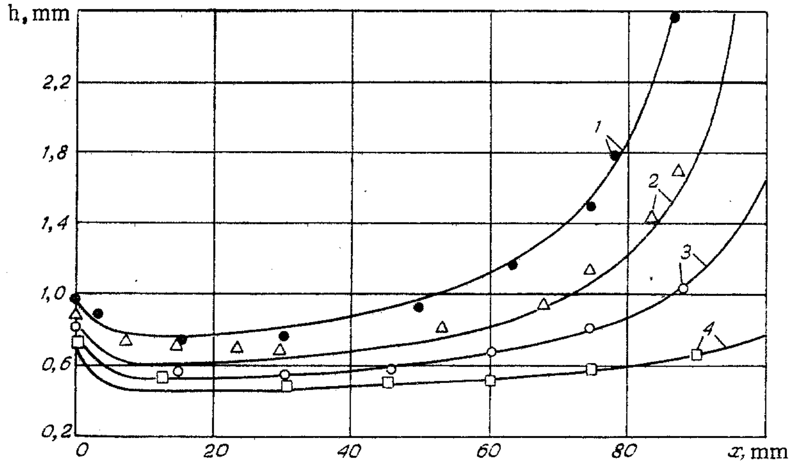

Fig. 5

As $x \to 1$ the first term in the right side of (16) increases without limit; hence it is valid until the film thinness condition is satisfied: $H/(1 - x) \ll 1/\varepsilon$ (this condition is satisfied for the experimental data presented).

Therefore, a two-dimensional model of fluid gravitational film flow over the reducer surface obtained on the basis of the boundary layer equations satisfactorily describes the flow in the fundamental section of the reducer, and can be used to compute heat- and mass-transfer processes in reducer-film apparatus.

The authors are grateful to V. E. Nakoryakov for attention to the research and for stimulating remarks, and to P. I. Geshev for aid in formulating the problem and in discussing the results.

LITERATURE CITED

1. A. M. Te, L. I. Sen', and A. M. Podsushnyi, "High-temperature heating of sea water in reducer-film apparatus," Ship Power Plants [in Russian], Izd. Dal'nyi Vostoknyi Gosudarst. Univ., Vladivostok (1981).
2. A. M. Te and L. I. Sen', "Fluid film flow over an reducer surface," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 3, Issue 1 (1983).
3. L. G. Loitsyanskii, Mechanics of Fluids and Gases [in Russian], Nauka, Moscow (1970).